OPTIMIZATION OF BURR PARAMETERS IN DRILLING USING STEP DRILLS

V.Durga Prasada Rao¹, T.V.Subba Rao², K.Hari Krishna³

¹Department of Mechanical Engineering, Asst. Professor S.R.K.R. Engineering College Bhimavaram-534 204, 
²Department of Mechanical Engineering, Professor S.R.K.R. Engineering College; Bhimavaram-534 204 
³Department of Mechanical Engineering, Student of M.E (CAD/CAM), S.R.K.R. Engineering College, 
Bhimavaram-534 204. 
¹Email: vdp_9@yahoo.co.in.

Abstract

Burr are sharp projections on the workpiece left uncut while machining. Many studies have been conducted to understand the burr formation mechanism. Concept of step drills is the latest addition to these studies, which is supposed to reduce burr height and thrust force. Most real world search and optimization problems naturally involve multiple objectives. Drilling problems do involve multiple objectives. In the present work, we discussed about the multi-objective optimization problem of minimizing burr height and thrust force in drilling using HSS step drills. The variables under consideration are step parameters –step angle, step size and step length. The weighted-sum approach is used in solving the multi-objective problem. The results are obtained by a direct search method, viz. Sequential Quadratic Programming (SQP) technique, and an indirect search method, viz. Penalty function method. The optimum values of feed and diameter of step drill are determined by solving the regression equations developed for them. The general method of iterative technique is used in solving the 2 second order nonlinear equations. The results given by the two methods are compared and they are found to be effective.

Key Words: Drilling, Optimal Design, Penalty Function, Quadratic Programming.

1.0 Introduction

Most machining processes produce burrs as a result of plastic deformation. The term burr is used to indicate the presence of unwanted material at work piece edges, which was not there prior to machining operations. This material sometimes appears as a short mound of material and sometimes as a long thin projection. There are two approaches in burr problem solution

- Minimizing the burr formation to reduce the cost of deburring and maintain part precision. This is a direct approach used in most machining processes.
- When minimization of burr formation cannot satisfy the product requirements, deburring is also necessary.

Many researchers experimentally studied the mechanism of burr formation in drilling operation and they have suggested optimum selection of cutting conditions, modification of tool and work piece geometry and proper selection of work materials to minimize burr size. Some of the review papers are Kim et al.[5], Kim, J., Dornfeld, D.A.[6], Lee, J.K., Sung-Lim Ko.[7], Lin, T.R., Shyu, R.F.[8], Min, S., Dornfeld, D.A.[9], Sung-Lim Ko.[12], Saunders.[13], and Xia, R.S., Madhavan, M.S.[14].

Ramu. C [11] has done some experimental investigations on minimization of burr size in drilling using step drills. Experiments were carried out by conventional twist drills to find the range of significant factors, namely feed and speed in which burr size is maximum. Two sets of experiments were conducted using step drills to optimize the step parameters so as to produce minimum burr height, and thrust force. Regression models were developed for burr height, and thrust force in terms of step parameters. Also regression models were developed for burr height, and thrust force in terms of feed and diameter of drill bit. All the models were developed by using Design-Expert software. Later optimum step parameters are determined with respect to each of the objectives, viz., Burr height and Thrust force. Finally comparative study was performed to prove the superiority of the step drills over conventional drills in reducing burr size.

In the present work, we deal with the multi-objective optimization problem of minimizing burr height, and thrust force in drilling using HSS step drills. The methodology involves the optimization of step parameters - step angle, step size and step length, and also to find the feed and diameter of step drill corresponding to optimal step parameters. The weighted - sum approach [2] is used in solving the multi-objective problem. The results are obtained by two different methods, viz., SQP method and penalty function method.
2.0 Problem Formulation

In first stage, two objectives, viz., Burr height and Thrust force are defined using the regression equations developed for the HSS step drills [11]. The variables under consideration are step parameters - step angle, step size and step length. The two objectives are:

Burr height, \( B = +33.13135 - (0.32725 \cdot A) - (24.16013 \cdot S) - (1.45638 \cdot L) + (2.84421 \cdot S^2) + (0.068388 \cdot L^2) + (0.20900 \cdot A \cdot S) + (0.00900 \cdot A \cdot L) \)

Thrust force, \( T = +34606.19103 - (336.93925 \cdot A) - (24380.69437 \cdot S) - (972.94012 \cdot L) + (2608.11179 \cdot S^2) + (35.17949 \cdot L^2) + (216.903 \cdot A \cdot S) + (7.51425 \cdot A \cdot L) \)

The method of objective weighing which is one of the classical methods of handling the multi-objective optimization problem [10], is used here to combine the two objectives, that are burr height (B) and thrust force(T). Thus, the weighted objective of the defined problem can be stated mathematically as:

Minimize : \( f = [w \cdot B] + [(1-w) \cdot T] \)

Variable bounds:
- \( 75^0 \leq \text{step angle} \leq 85^0 \)
- \( 1\text{mm} \leq \text{step size} \leq 2\text{mm} \)
- \( 3\text{mm} \leq \text{step length} \leq 7\text{mm} \)

In the second stage, Regression models developed by ‘design of experiments methodology’ for two responses namely burr height and thrust force are used to find the diameter (D) and feed (F) corresponding to optimum step parameters. The two responses - burr height and thrust force in terms of feed and outer diameter of HSS step drills are given by:

Burr height = \( - 4.87689 + (0.76408 \cdot D) - (1.36095 \cdot F) - (0.027888 \cdot D^2) - (10.45039 \cdot F^2) + (0.41071 \cdot D \cdot F) \)

Thrust force = \( +5460.77496 - (797.82563 \cdot D) + (4118.48950 \cdot F) + (33.01182 \cdot D^2) - (26480.17945 \cdot F^2) + (878.57679 \cdot D \cdot F) \)

The two equations are solved for feed and diameter by using the program of the general method of iterative technique [4].

3.0 Penalty Function Method

Penalty function method transforms the constrained optimization problem into a sequence of unconstrained problems by adding penalty terms for each constraint violation. The penalty function formulations can be of two categories: interior and exterior methods. In the interior formulations, some popularly used penalty terms are inverse penalty and log penalty. Some commonly used penalty terms in the case of exterior penalty function formulations are infinite barrier and bracket operator penalty [10].

The algorithm [3] for a basic optimization problem which minimizes \( f(x) \) subject to \( g_j (x) \geq 0, j=1,2,\ldots,J \), and \( h_l (x) = 0, l=1,2,\ldots,L \) is given hereunder.

**Step-1:** Choose two termination parameters \( \varepsilon_1, \varepsilon_2 \), an initial solution \( x^{(0)} \), a penalty term \( \Omega \) and an initial penalty parameter \( R^{(0)} \). Chose a parameter \( c \) to update \( R \) such that \( 0 < c < 1 \) is used for interior penalty terms and \( c>1 \) is used for exterior penalty terms. Set \( t = 0 \).

**Step-2:** Form the penalty function \( P[x^{(t)}, R^{(t)}] = f(x^{(t)}) + \Omega [R^{(t)}, g(x^{(t)}), h(x^{(t)})] \).

**Step-3:** Starting with a solution \( x^{(0)} \), find \( x^{(t+1)} \) such that \( P[x^{(t+1)}, R^{(t)}] \) is minimum for a fixed value of \( R^{(t)} \). Use \( \varepsilon_1 \) to terminate the unconstrained search.

**Step-4:** Is \( | P [x^{(t+1)}, R^{(t)}] - P[x^{(t)}, R^{(t)}] | \leq \varepsilon_2 ? \) If yes, set \( x^{(t)} = x^{(t+1)} \) and terminate; else go to step-5.

**Step-5:** Choose \( R^{(t+1)} = cR^{(t)} \). Set \( t = (t + 1) \) and go to step-2.

The optimum solution to the numerical problem is found by using a computer program that incorporates a bracket operator penalty term \( [\Omega = R (g(x))^2] \) in the penalty function. It uses steepest descent method for multivariable function optimization which uses a combination of the bounding phase and the golden section search to achieve a uni-directional search.
4.0 Sequential Quadratic Programming

Sequential quadratic programming (SQP) methods have received a lot of attention in recent years owing to their superior rate of convergence. The method has a theoretical basis that is related to (a) the solution of a set of non-linear equations using Newton’s method, and (b) the derivation of a non-linear equation using Kuhn-Tucker conditions to the lagrangian of the constrained optimization problem.

As with all gradient methods, there are two tasks: direction finding in the design space, and step size selection [1].

Consider a non-linear optimization problem:

Find \( X \) which minimizes \( f(x) \)
Subject to: \( g_i(x) \leq 0, \quad i = 1, 2, 3 \ldots m \)
\( h_j(x) = 0, \quad j = 1, 2, 3 \ldots l \)

Then the QP sub-problem to find the direction vector \( d_k \) is given by:

Minimize \( \frac{1}{2} d^T d + f^T d \)
Subject to
\( g_i^T d + g_i^k \leq 0, \quad i \in I_1 \)
\( h_j^T d + h_j^k \leq 0, \quad j \in I_2 \)
\( x^L \leq d + x^k \leq x^U \)

Note that \( d_i, i=1, 2 \ldots, n \) are variables in this QP. We will denote the solution to this QP as \( d_k \).

The gradient vectors \( f, g, \) and \( h \) are evaluated at \( X_k \). The sets \( I_1, I_2 \) are active sets – not all constraints in the original problem enter into the QP sub-problem. Specifically, the active sets are defined as:

\( I_1 = \{ j: g_j(X_k) \geq V(X_k) - \delta, j = 1, 2, \ldots, m \} \)
\( I_2 = \{ j: \vert h_j(X_k) \vert \geq V(X_k) - \delta, j = 1, 2, \ldots, m \} \)

Where \( \delta \) is a small number specified by the user and \( V \) represents the maximum violation as defined by

\( V(X_k) = \max \{ \vert h_j(X_k) \vert, \quad j = 1, 2, \ldots, m \} \)

Furthor no active set strategy is to be used for the bounds. Thus the solution \( d_k \) must satisfy KKT conditions to the problem:

Minimize \( \frac{1}{2} d^T d + f^T d \)
Subject to
\( [A]d + \hat{g} = 0 \)

Where the rows of matrix \( A \) are the gradients of the constraints in \( \hat{T} \) and \( \hat{g} \) value of the constraints in the set.

Assuming rows are linearly independent, we have:

Lagrangian, \( L = \frac{1}{2} d^T + \nabla f^T d + \mu^T (A d + g^\hat{\}) \). Where \( \mu \) is set of Lagrange multipliers.

For minimizing \( L \), setting \( \frac{\partial L}{\partial d} = 0 \),

We get \( d = -(-A^T \mu - P f) \)
And \( \mu = (AA^T)^{-1}(-A^T f + Ag^\hat{}) \) ..............................................(11)

This allows us to write: \( d = d^f + d^g \).

Where \( d^f \) is projection of steepest descent direction.
\( = -P [\nabla f \] where \( [P] = I - A^T [AA^T]^T A \)
\( d^g \) is corrective step towards constraint region
\( = A^T [AA^T]^T g^\hat{} \)

Since \( d \) is a combination of cost reduction step \( d^f \) and constraint violation reduction step \( d^g \), we can expect the function:

\( \theta(x) = f(x) + R V(x) \) .............................................................................(12)

To reduce along \( d \) provided the penalty parameter \( R \) is sufficiently large. Now the procedure to find the step \( \alpha \) is based on approximate line search strategy as follows.

The step \( \alpha \) is chosen equal to \( (0.5)^j \), where \( j \) is the first of the integers \( q=0,1,2 \ldots \) for which the following inequality holds:

\( \theta(X_k + (0.5)^q d_k) \leq \theta(X_k) - \gamma (0.5)^q \quad \|d\| \leq \alpha \) .............................................................................(13)

Once the \( \alpha \) is determined, a new point obtained from \( X_{k+1} = X_k + \alpha d_k \). We reset \( X_k = X_{k+1} \) and again solve the QP equations (2) for a new direction vector.

Stopping criteria:
If \( d_k = 0 \) is the solution to the QP in equations (2), then we can show that \( X_k \) is a KKT point \( [i.e., \quad \|d\| \leq TOL \alpha] \).
In addition, the iterative process is terminated if there is no change in \( f \) for three consecutive iterations (with \( V \leq \) TOL).
Algorithm:
1. Choose a starting point \( X_0 \) which satisfies the bounds \( X^L \leq X_0 \leq X^U \). Select scalars \( \delta, R, \gamma \).
2. Determine the active set, evaluate gradients of active constraints and solve the QP in equations (2) to obtain \( d_k \).
3. Check for convergence.
4. Determine the step size \( \alpha_k \) from equation (12) update the design point as \( X_{k+1} = X_k + \alpha_k d_k \). We reset \( X_k = X_{k+1} \) and return to step(2).

5.0 Results
In the first stage, the optimum solution is found by two methods, viz., SQP method and penalty function method. The input parameters to run the computer program of penalty function method are:
Parameter 'C' to update \( R = 10 \) for this problem.
Initial value of Penalty parameter \( R = 0.001 \)
Initial vector = \([75 1 3]^T\) for this problem.

In SQP method, the solution is obtained by solving simulation equations derived from Kuhn-Tucker conditions. The scalars used to run the computer program are: Delta \( (\delta) = 0.01 \), penalty parameter \( (R) = 100.0 \), Gamma \( (\gamma) = 0.1 \). The optimum solutions (convergence-KKT points) are obtained after the no. of iterations varying from 142 to 269.

The optimal solutions obtained by C-program written for Penalty function and SQP methods are given in table-1 and table-2. Figure.1 shows the Pareto optimal front \([w*B] \) versus \([(1-w)*T] \) obtained for both methods (table.2).

In the second stage, the feed and diameter are determined by a computer program of general method of iterative technique. The results are shown in table.1.

<table>
<thead>
<tr>
<th>Weight (W)</th>
<th>Initial Vector: Step angle Step size Step length</th>
<th>PENALTY FUNCTION</th>
<th>SEQUENTIAL QUADRATIC PROGRAMING</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.047961</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.2</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.040132</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.3</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.047472</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.4</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.046213</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.5</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.049664</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.6</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.043456</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.7</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.042209</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.8</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.044017</td>
<td>137.37.136</td>
</tr>
<tr>
<td>0.9</td>
<td>( 75 \ 0.00525 \ 1.50121 \ 0.00690 )</td>
<td>0.040354</td>
<td>137.37.136</td>
</tr>
</tbody>
</table>
Table 2 Comparison between penalty method and sequential quadratic programming

<table>
<thead>
<tr>
<th>Weight (W)</th>
<th>Penalty function method</th>
<th>SQP method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total weighted Objective</td>
<td>Total weighted Objective</td>
</tr>
<tr>
<td>(W)</td>
<td>(w * E)</td>
<td>((1-w) * T)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.004766</td>
<td>1653.477</td>
</tr>
<tr>
<td>0.2</td>
<td>0.009626</td>
<td>1469.905</td>
</tr>
<tr>
<td>0.3</td>
<td>0.014242</td>
<td>1286.024</td>
</tr>
<tr>
<td>0.4</td>
<td>0.018486</td>
<td>1102.097</td>
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<tr>
<td>0.5</td>
<td>0.022482</td>
<td>918.236</td>
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<tr>
<td>0.6</td>
<td>0.026074</td>
<td>734.4535</td>
</tr>
<tr>
<td>0.7</td>
<td>0.029602</td>
<td>550.7624</td>
</tr>
<tr>
<td>0.8</td>
<td>0.032275</td>
<td>367.1307</td>
</tr>
<tr>
<td>0.9</td>
<td>0.03631</td>
<td>183.6654</td>
</tr>
</tbody>
</table>

**6.0 Conclusions**

The multi objective optimization problem solved for optimum step parameters (step angle, step size, and step length) of HSS step drills shows that, to get minimum burr height and thrust force, step diameter should be approximately 70% of drill diameter and step length should be about 60% of drill diameter. It is better to have reasonably large step angle of the order of 75°. These studies revealed that minimum burr height and thrust force can be obtained even for large diameter step drills by using optimized step parameters. These drill bits can be run at reasonably high feeds to give minimum burr height and thrust force at which conventional drill bits can’t be run.

In this work, we have taken limited range of parameters only. Regression models developed can be applicable within this range only and more studies have to be conducted to properly understand the relationships between the responses and variables. Also in this work, only two objective functions consisting of three variables are taken without any constraints. However, Burr formation also depends on other variables such as torque, temperature generated during drilling, point angle, helix angle, work material properties, work material geometry…etc. These should also be considered to have the clear picture of burr formation.
References